The BEC-BCS crossover from the normal to the superfluid phase

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We will explore the properties of a two-component gas of atomic fermions, with an emphasis on the dynamical properties such as transport, dissipation and collective modes. Conceptually, the two-component Fermi gas is one of the simplest and purest quantum many-body system ever realized, with nowadays nearly homogeneous trapping geometries [1], a complete absence of disorder, and interatomic interactions characterized only by the scattering length a [2]. At the same time, the system displays a remarkably rich phase diagrams, which forces us to summon very different approaches of the quantum many-body problem. It also allows us to draw fruitful analogies with other fermionic or bosonic fluids, such as electrons gases, superfluid Helium or nuclear matter.

As a function of temperature, the gas evolves from a macroscopically-coherent superfluid phase at low temperature, to a normal quantum gas and eventually to a classical gas at high temperature. As a function of the interaction strength, it displays a well-known transition from a fermionic to a bosonic physics: the Bose-Einstein Condensate to Bardeen-Cooper-Schrieffer crossover, which (despite its name) applies to both the normal and superfluid phases [3].

During the first lecture, we will explore the low-temperature fermionic side of this crossover. Above the critical temperature T_c and much below the Fermi temperature $T_{\rm F}$, the gas behaves as a gas of Landau quasiparticles: a Fermi liquid [4]. From the properties of the Landau quasiparticles, I will explain how to derive the Naviers-Stokes equations obeyed by the hydrodynamic perturbations [5, 6]. I will also discuss the existence of collisionless collective modes. When approaching T_c from above, a soft pairing mode emerges in the quasiparticle gas, which we will interpret as a precursor of the superfluid phase transition.

During the second lecture, we will venture below T_c. The Landau quasiparticles then turn into BCS quasiparticles, with a gapped spectrum [7]. This drastically modifies the collective physics, with the emergence of an Anderson-Bogoliubov phase mode at low energy, and an amplitude Higgs mode at high energy, in place of the density modes of the Fermi liquid. Leaving the fermionic side of the phase diagram, we will study the dispersion of these two modes along the BCS-BEC crossover [8–11]. In the regime $T \ll T_c$, the phononic depart of the Anderson-Bogoliubov branch defines a universal regime where the dynamics of the gas is entirely determined by its phononic degrees of freedom [12, 13]. We will discuss the dissipative properties of this superfluid of phonons.

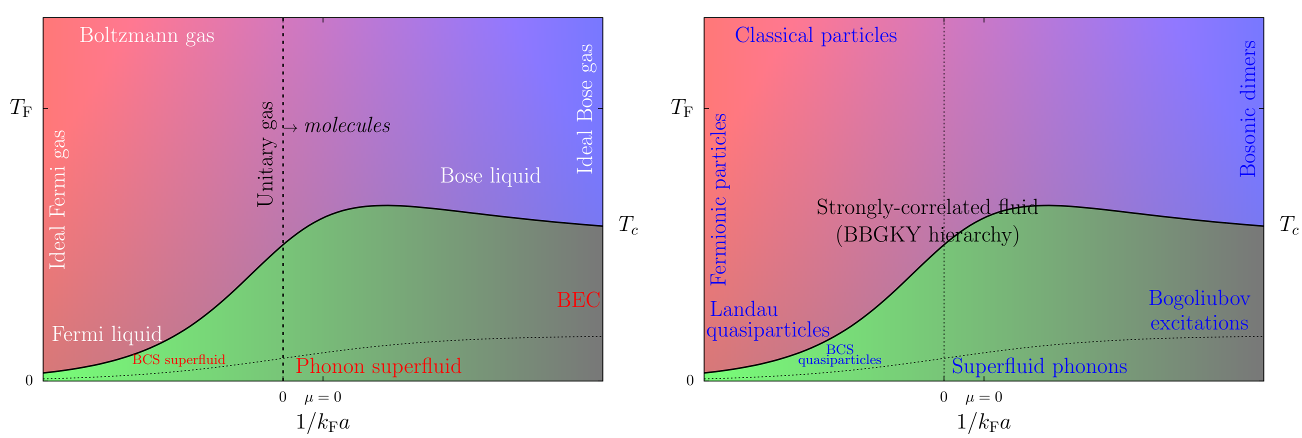

FIG. 1. (Left panel) Equilibrium phase diagram of the spin-1/2 Fermi gas with contact interactions. The various regimes of the normal and superfluid phase are shown in white and red respectively. (Right panel) Same diagram showing (in blue) the microscopic degrees of freedom that underly the dynamical properties.

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